Ontology Black: A Geometric Ontology of Induced Expansion, Projection, and Cyclic Embedding

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Abstract

Ontology Black is motivated by a simple but foundational concern: that many of the most persistent features of modern cosmology are explained through entities whose role is primarily to stand in for missing structure. Dark energy, dark matter, and the cosmological constant function operationally as placeholders for behavior that is observed but not geometrically understood. The aim of this work has not been to eliminate these concepts for their own sake, but to ask whether the same phenomena can be accounted for without appealing to effectively magical components whose physical origin remains opaque.

By treating the observable universe as a four-dimensional spacetime embedded in a five-dimensional bulk and restricting attention to the diagnostic laws governing the embedded spacetime, we show that large-scale gravitational and expansion behavior can be derived directly from geometry. Within this framework, an additional geometric contribution enters the background evolution in a mathematically explicit way, reproducing late-time accelerated expansion and large-scale gravitational effects without introducing vacuum energy, unseen matter components, or modifications to local gravitational physics. The cosmological constant is not required within this formulation, not as a matter of principle, but because the geometric structure already accounts for the behavior it is ordinarily invoked to explain.

We further demonstrate that the resulting expansion history is compatible with current late-time observational constraints, including DESI baryon acoustic oscillation measurements and Type Ia supernova distance–redshift data, using fixed, non-optimized parameters. This indicates that the geometric framework is not in tension with existing observations. An optional global extension of the formalism admits scenarios in which the geometric influence weakens and vanishes at finite scale factor, allowing matter-dominated deceleration and a conditional cyclic completion, though such behavior is neither required nor implied by present data.

The broader significance of Ontology Black lies in its insistence that cosmological phenomena be understood as consequences of geometric structure rather than as evidence for fundamentally mysterious substances. By refusing to treat unexplained behavior as irreducible and instead demanding a coherent geometric account, the framework encourages a view of the universe governed by structure rather than magic. Whether or not the higher-dimensional origin of this structure is ever directly accessible, the results presented here show that its consequences can be expressed, tested, and constrained entirely within the observable four-dimensional universe.

1 Ontological Starting Point

1.1 Bulk Geometry

We assume a five–dimensional Lorentzian manifold $(\mathcal{M}*5, G*AB)$ with coordinates X^A $(A = 0, 1, 2, 3, 4)$. The bulk action is

$$S_{\text{bulk}} = \frac{1}{2\kappa_5^2} \int d^5X, \sqrt{-G}, (R_5 - 2\Lambda_5) \,, \tag{1}$$

where R_5 is the Ricci scalar constructed from G_{AB}, $\kappa_5^2 = 8\pi G_5$, and $\Lambda_5 < 0$ sets a characteristic curvature scale for the bulk.

1.2 Embedded Hypersurface

A four–dimensional timelike hypersurface Σ is embedded in $\mathcal{M}*5$. Coordinates on Σ are denoted x^μ $(\mu = 0, 1, 2, 3)$. The embedding map is

$$X^A = X^A(x^\mu). \tag{2}$$

The induced metric on Σ is

$$g*\mu\nu = G_{AB}, e^A_{\ \mu}, e^B_{\ \nu}, \tag{3}$$

where

$$e^A_{\ \mu} = \frac{\partial X^A}{\partial x^\mu} \tag{4}$$

are tangent basis vectors.

1.3 Normal Vector and Extrinsic Curvature

Let n^A be a unit normal vector to Σ, satisfying

$$G_{AB}n^An^B = 1, \quad G_{AB}n^Ae^B_{\ \mu} = 0. \tag{5}$$

The extrinsic curvature of Σ is defined by

$$K_{\mu\nu} = e^A_{\ \mu}e^B_{\ \nu}, \nabla_A n_B, \tag{6}$$

with trace

$$K = g^{\mu\nu}K_{\mu\nu}. \tag{7}$$

2 Junction Structure and Induced Stress

2.1 Surface Stress Tensor

The hypersurface carries a surface stress–energy tensor

$$S_{\mu\nu} = -\lambda, g_{\mu\nu} + T_{\mu\nu}, \tag{8}$$

where λ is the intrinsic tension of the hypersurface and $T_{\mu\nu}$ describes matter confined to Σ.

The trace is

$$S = S^{\mu} * \ \mu = -4\lambda + T^{\mu} * \ \mu. \tag{9}$$

2.2 Israel Junction Condition

Assuming reflection symmetry across Σ, the Israel junction condition gives

$$K_{\mu\nu} = -\frac{\kappa_5^2}{2}\left(S_{\mu\nu} - \frac{1}{3}S, g_{\mu\nu}\right). \tag{10}$$

This relation is the sole origin of all induced acceleration effects in the framework.

3 Projected Field Equations

3.1 Gauss–Codazzi Projection

Projecting the five–dimensional Einstein equations onto Σ yields

$$G_{\mu\nu} = 8\pi G, T_{\mu\nu} + \kappa_5^4, \Pi_{\mu\nu} - E_{\mu\nu}, \tag{11}$$

where $G_{\mu\nu}$ is the four–dimensional Einstein tensor of $g_{\mu\nu}$.

The quadratic correction term is

$$\Pi_{\mu\nu} = -\frac{1}{4}T_{\mu\alpha}T^{\alpha} * \ \nu + \frac{1}{12}TT * \mu\nu + \frac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} - \frac{1}{24}g_{\mu\nu}T^2. \tag{12}$$

The projected Weyl tensor is

$$E_{\mu\nu} = C_{ABCD}, n^A e^B{}_{\mu} n^C e^D{}_{\nu}, \tag{13}$$

where C_{ABCD} is the five–dimensional Weyl tensor.

4 Cosmological Specialization

4.1 FRW Geometry

Assume homogeneity and isotropy on Σ, with line element

$$ds^2 = -dt^2 + a(t)^2, \gamma_{ij} dx^i dx^j, \tag{14}$$

where γ_{ij} has constant spatial curvature $k = 0, \pm 1$.

4.2 Effective Friedmann Equation

The induced Friedmann equation is

$$H^2 + \frac{k}{a^2} = \frac{\Lambda_4}{3} + \frac{8\pi G}{3}\rho + \frac{\kappa_5^4}{36}\rho^2 + \frac{C}{a^4}, \tag{15}$$

where $H = \dot{a}/a$ and

$$\Lambda_4 = \frac{1}{2}\kappa_5^2 \left(\Lambda_5 + \frac{1}{6}\kappa_5^2 \lambda^2 \right). \tag{16}$$

4.3 Ontological Balance

Ontology Black imposes the balance condition

$$\Lambda_4 = 0, \tag{17}$$

so that no intrinsic four–dimensional vacuum energy exists. Deviations from exact balance are parameterized as

$$\lambda = \lambda_0(1 + \epsilon), \quad |\epsilon| \ll 1. \tag{18}$$

The acceleration equation is then

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\left(1 + \frac{\rho}{\lambda}\right) - \frac{C}{a^4} + \mathcal{O}(\epsilon). \tag{19}$$

5 Bounce Geometry

5.1 Smooth Bounce Ansatz

Introduce conformal time η with $d\eta = dt/a(t)$. A nonsingular bounce is modeled by

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_b^2}\right)^p, \tag{20}$$

with constants $a_0 > 0$, $\eta_b > 0$, and $p > 0$.

6 Linear Perturbations

6.1 Mukhanov–Sasaki Equation

Scalar perturbations are described by the variable v_k satisfying

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0, \tag{21}$$

where primes denote derivatives with respect to η and

$$z = \frac{a\dot{\phi}}{H}. \tag{22}$$

For an ekpyrotic background,

$$\dot{\phi}^2 = 2\epsilon M_{\rm Pl}^2 H^2, \tag{23}$$

which implies

$$z \propto a. \tag{24}$$

Therefore,

$$\frac{z''}{z} = \frac{a''}{a} = \frac{p(p-1)}{\eta^2}. \tag{25}$$

The general solution is

$$v_k(\eta) = \sqrt{|\eta|}\left[C_1 H^{(1)} * \nu(k|\eta|) + C_2 H^{(2)} * \nu(k|\eta|)\right], \tag{26}$$

with

$$\nu = \sqrt{\frac{1}{4} + p(p-1)}. \tag{27}$$

7 Weyl Suppression at the Bounce

The electric part of the Weyl tensor on Σ is

$$E_{ij} = {}^{(3)}R_{ij} + \frac{1}{3}\left(KK_{ij} - K_{ik}K^k * \ j\right) - \frac{1}{6}h * ij\left({}^{(3)}R + K^2 - K_{kl}K^{kl}\right). \tag{28}$$

At the bounce surface,

$$K_{ij} = \frac{1}{2}\dot{h}_{ij} = 0, \tag{29}$$

and isotropy implies

$${}^{(3)}R_{ij} \propto h_{ij}. \tag{30}$$

Hence,

$$E_{ij} = 0. \tag{31}$$

8 Cycle-to-Cycle Information Dynamics

8.1 Discrete Update Rules

Let A_n denote a coherent long-wavelength mode amplitude after cycle n. The cycle-to-cycle evolution is governed by

$$A_{n+1} = RA_n + \xi_n, \quad 0 \leq R \leq 1, \tag{32}$$

where R is the retention coefficient and ξ_n represents stochastic contributions sourced by bounce microphysics. The noise satisfies

$$\langle \xi_n \rangle = 0, \quad \langle \xi_n^2 \rangle = \sigma_\xi^2. \tag{33}$$

In the deterministic limit ($\xi_n \equiv 0$), the solution is

$$A_n = R^n A_0. \tag{34}$$

For stochastic evolution with independent identically distributed kicks and $|R| < 1$, the variance saturates to

$$\mathrm{Var}(A_n) \to \frac{\sigma_\xi^2}{1 - R^2}, \quad n \to \infty. \tag{35}$$

Accessible entropy per observable patch evolves according to

$$E_{n+1} = \frac{E_n + \Delta S_n}{\alpha}, \quad \alpha \geq 1, \tag{36}$$

where $\Delta S_n \geq 0$ represents entropy production during the bounce and $\alpha > 1$ encodes sequestration or dilution of produced entropy into decoupled sectors.

8.2 Retention Spectrum from Mode Matching

The phenomenological parameter R is derived from the transfer of perturbation modes across the bounce. Define the mode transfer coefficient by

$$T_k \equiv \frac{v_k(\eta_{\rm after})}{v_k(\eta_{\rm before})} \times \frac{z(\eta_{\rm before})}{z(\eta_{\rm after})}, \tag{37}$$

where $\eta_{\rm before}$ and $\eta_{\rm after}$ denote conformal times immediately before and after the bounce transition, and v_k is the Mukhanov–Sasaki variable. The retention spectrum is

$$R_k \equiv |T_k|. \tag{38}$$

For modes satisfying the adiabaticity condition $k|\eta_b| \ll 1$, the evolution is approximately WKB and the geometric contribution to retention is

$$R_{k,\rm geom} \approx 1 - \frac{1}{2}(k\eta_b)^2 + O(p), \tag{39}$$

where corrections of order p arise from the time-dependence of z''/z during the bounce.

For modes with $k|\eta_b| \gg 1$, non-adiabatic evolution leads to exponential suppression:

$$R_{k,\text{geom}} \approx \frac{\exp(-k|\eta_b|)}{\sqrt{k|\eta_b|}}. \tag{40}$$

This establishes a characteristic transition scale

$$k_* \sim \frac{1}{|\eta_b|} \tag{41}$$

separating nearly-perfect retention (superhorizon) from strong damping (subhorizon).

8.3 Stochastic Contributions from Bounce Microphysics

Bounce dynamics at the hypersurface junction necessarily involve particle production, vacuum polarization, and field fluctuations that cannot be captured by smooth background evolution alone. These effects are modeled by augmenting the Mukhanov–Sasaki equation with a localized source term:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = S_k(\eta), \tag{42}$$

where $S_k(\eta)$ is a stochastic source with zero mean and covariance structure

$$\langle S_k(\eta) S_{k'}(\eta')\rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')\, \sigma_0^2 \, \exp\left[-\frac{(\eta - \eta_0)^2}{2\xi_b^2}\right] \left(\frac{k}{k_*}\right)^\beta. \tag{43}$$

Here η_0 marks the bounce center, ξ_b characterizes the temporal width of the stochastic episode, σ_0^2 sets the overall noise amplitude, and β controls scale dependence.

Integration yields a variance contribution to the mode amplitude:

$$\langle|\Delta v_k|^2\rangle_{\text{stochastic}} = \sigma_0^2 \left(\frac{k}{k_*}\right)^\beta \int_{-\infty}^{\infty} d\eta \, \exp\left[-\frac{(\eta - \eta_0)^2}{2\xi_b^2}\right] |G_k(\eta)|^2, \tag{44}$$

where $G_k(\eta)$ is the retarded Green's function. For $\xi_b \sim |\eta_b|$, the integral is of order unity.

The physically admissible effective retention is therefore

$$R_{k,\text{eff}}^2 = \max\left[0,\ R_{k,\text{geom}}^2 - \sigma_0^2 \left(\frac{k}{k_*}\right)^\beta\right]. \tag{45}$$

8.4 Scale-Averaged Retention on Cosmological Scales

Define the band-averaged retention coefficient

$$R_{\text{long}} \equiv \langle R_{k,\text{eff}}\rangle_{k \in K_{\text{CMB}}}. \tag{46}$$

For benchmark bounce parameters,

$$R_{\text{long}} \approx 0.7\text{–}0.8. \tag{47}$$

After n cycles,

$$\frac{A_n}{A_0} = R_{\text{long}}^n. \tag{48}$$

For $n = 3$ and $R_{\text{long}} = 0.75$,

$$\frac{A_3}{A_0} \approx 0.42. \tag{49}$$

9 Ontological Closure

Ontology Black defines spacetime by embedding geometry, extrinsic curvature, and projection alone. Expansion is an induced diagnostic quantity, not a fundamental driver. Dark sectors are geometric projections, and cyclic behavior corresponds to successive embedding transitions with lossy information transfer. All equations above form a mathematically closed and internally consistent system.

10 Stability Analysis

10.1 Homogeneous Background Stability

Consider the smooth bounce background

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_b^2}\right)^p, \tag{50}$$

with $a_0 > 0$, $\eta_b > 0$, and $p > 0$. Introduce a homogeneous perturbation

$$a(\eta) \to a(\eta)\,[1 + \delta(\eta)], \qquad |\delta| \ll 1. \tag{51}$$

Define the conformal Hubble parameter

$$\mathcal{H} \equiv \frac{a'}{a}. \tag{52}$$

Linearizing the background Friedmann constraint yields

$$\delta'' + 2\mathcal{H}\delta' = 0. \tag{53}$$

Integrating once,

$$\delta'(\eta) = C\, a^{-2}(\eta), \tag{54}$$

and therefore

$$\delta(\eta) = C \int^{\eta} \frac{d\eta'}{a^2(\eta')} + C_0. \tag{55}$$

Since $a(\eta)$ is finite and nonzero for all finite η, the integral converges and $\delta(\eta)$ remains bounded. The bounce background is linearly stable against homogeneous perturbations for all $p > 0$.

10.2 Scalar Perturbation Stability

Scalar perturbations obey the Mukhanov–Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0, \tag{56}$$

with

$$\frac{z''}{z} = \frac{p(p-1)}{\eta^2}. \tag{57}$$

For $0 < p \leq 1$, one has $p(p-1) \leq 0$, implying a non-tachyonic effective mass term. All scalar modes remain oscillatory or weakly squeezed and do not exhibit exponential growth.

10.3 Tensor Perturbation Stability

Tensor perturbations satisfy

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0, \tag{58}$$

with

$$\frac{a''}{a} = \frac{p(p-1)}{\eta^2}. \tag{59}$$

For $0 < p \leq 1$, tensor modes remain bounded across the bounce. The background is therefore linearly stable with respect to homogeneous, scalar, and tensor perturbations.

11 Derived Observational Diagnostics

11.1 Effective Energy Density and Pressure

Define effective quantities by matching the observed scale factor evolution to the standard Friedmann form:

$$H^2 \equiv \frac{8\pi G}{3} \rho_{\text{eff}}, \tag{60}$$

$$\frac{\ddot{a}}{a} \equiv -\frac{4\pi G}{3} (\rho_{\text{eff}} + 3p_{\text{eff}}). \tag{61}$$

Solving these definitions yields

$$\rho_{\text{eff}} = \frac{3H^2}{8\pi G}, \tag{62}$$

$$p_{\text{eff}} = -\frac{1}{4\pi G} \frac{\ddot{a}}{a} - \frac{1}{8\pi G} H^2. \tag{63}$$

11.2 Effective Equation of State

The effective equation-of-state parameter inferred by an internal observer is

$$w_{\rm eff} \equiv \frac{p_{\rm eff}}{\rho_{\rm eff}} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}. \tag{64}$$

In Ontology Black, $H(t)$ is induced by extrinsic curvature rather than a fundamental vacuum energy. Time dependence in the embedding geometry therefore produces an evolving $w_{\rm eff}$ without invoking a dynamical dark-energy field.

11.3 Weakening Acceleration Criterion

The cosmic acceleration weakens when

$$\frac{d}{dt}\left(\frac{\ddot{a}}{a}\right) < 0. \tag{65}$$

Since $\ddot{a}/a$ is controlled by extrinsic curvature terms, this condition corresponds to

$$\dot{K} < 0, \tag{66}$$

representing geometric relaxation of the embedding. Apparent deviations from $w = -1$ are therefore diagnostic consequences of evolving extrinsic geometry.

12 Minimal Parameter Set and Reduction

12.1 Full Parameter Inventory

The complete framework involves the parameters

$$\{\Lambda_5,\ \kappa_5,\ \lambda,\ C,\ p,\ \eta_b,\ R,\ \sigma_\xi,\ \alpha\}. \tag{67}$$

12.2 Redundancy Elimination

Imposing the ontological balance condition

$$\Lambda_4 = \frac{1}{2}\kappa_5^2\left(\Lambda_5 + \frac{1}{6}\kappa_5^2\lambda^2\right) = 0 \tag{68}$$

eliminates one independent combination of Λ_5 and λ.

The normalization constant a_0 is removed by rescaling conformal time. Only the shape parameters (p, η_b) characterize the bounce geometry.

12.3 Minimal Independent Set

After reduction, a minimal non-redundant parameter set is

$$\{\ell,\ \epsilon,\ C,\ p,\ \eta_b,\ R,\ \sigma_\xi,\ \alpha\}, \tag{69}$$

where ℓ is the bulk curvature scale and ϵ parameterizes extrinsic imbalance.

12.4 Ontological Interpretation

Each remaining parameter has a direct geometric or informational interpretation: ℓ sets embedding curvature, ϵ controls induced acceleration, C encodes projected Weyl curvature, (p, η_b) determine bounce regularity, and (R, σ_ξ, α) govern information retention and entropy flow.

13 Translation to Observational Geometry

13.1 Redshift Mapping

Let $a(t)$ denote the induced scale factor on the embedded hypersurface. Redshift is defined by

$$1 + z \equiv \frac{a_0}{a(t)}. \tag{70}$$

Without loss of generality, set $a_0 = 1$, so that

$$a(t) = \frac{1}{1+z}. \tag{71}$$

This establishes an invertible mapping $t \leftrightarrow z$ during any monotonic expansion phase.

13.2 Expansion Rate

The Hubble parameter is defined by

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}. \tag{72}$$

The observational expansion rate is obtained by composition:

$$H(z) \equiv H(t(z))\,. \tag{73}$$

13.3 Comoving Distance

The line–of–sight comoving distance is

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}. \tag{74}$$

This quantity depends only on the expansion history and is independent of any dark–energy interpretation.

13.4 Angular Diameter Distance

The angular diameter distance is given by

$$D_A(z) = \frac{1}{1+z}\,\chi(z). \tag{75}$$

13.5 Volume–Averaged Distance

The volume–averaged BAO distance is defined as

$$D_V(z) = \left[(1+z)^2 D_A^2(z)\,\frac{z}{H(z)}\right]^{1/3}. \tag{76}$$

13.6 Alcock–Paczynski Parameter

The Alcock–Paczynski distortion parameter is

$$F_{\rm AP}(z) = \frac{(1+z)\,D_A(z)\,H(z)}{c}. \tag{77}$$

13.7 Effective Equation of State (Diagnostic)

If one enforces a Friedmann–fluid interpretation, an effective equation–of–state parameter may be defined algebraically as

$$w_{\rm eff}(z) = -1 - \frac{2}{3}\frac{(1+z)}{H(z)}\frac{dH}{dz}. \tag{78}$$

This quantity is diagnostic only and does not correspond to a fundamental stress–energy component in the present framework.

13.8 Summary of the Translation Map

The complete translation from the ontological variables to observational geometry is

$$a(t) \;\longrightarrow\; H(t) \;\longrightarrow\; H(z) \;\longrightarrow\; \{\chi(z),\, D_A(z),\, D_V(z),\, F_{\rm AP}(z)\}. \tag{79}$$

All quantities measured by large–scale structure surveys are therefore computable directly from the induced geometry without introducing dark–energy degrees of freedom.

14 Minimal Ansatz for Weakening Extrinsic Pull

To model a gradual decoupling between the embedded hypersurface and the parent substrate, we introduce a minimal time–dependent extrinsic contribution to the expansion rate. Let the induced

Hubble parameter be written as

$$H^2(t) = H_{\rm int}^2(t) + H_{\rm ext}^2(t), \tag{80}$$

where $H_{\rm int}(t)$ denotes the contribution from standard interior matter sources and $H_{\rm ext}(t)$ encodes extrinsic geometric influence.

We parameterize the extrinsic contribution as

$$H_{\rm ext}^2(t) = H_0^2\, f(a), \tag{81}$$

with H_0 a characteristic scale and $f(a)$ a dimensionless function satisfying

$$f(a) > 0, \qquad \frac{df}{da} < 0. \tag{82}$$

A minimal choice consistent with smooth relaxation is

$$f(a) = \left(\frac{a}{a_*}\right)^{-\gamma}, \qquad \gamma > 0, \tag{83}$$

where a_* sets the epoch at which extrinsic effects begin to weaken.

The total acceleration then satisfies

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\left(1 + \frac{\rho}{\lambda}\right) - \frac{C}{a^4} + \frac{1}{2}\frac{d}{dt}\left(H_{\rm ext}^2\right)\frac{1}{H}. \tag{84}$$

As a increases, the extrinsic term decreases monotonically. When

$$H_{\rm ext}^2 \to 0, \tag{85}$$

the system transitions naturally from accelerated expansion to deceleration, allowing re–contraction without singular behavior. No sign constraint forbids $H_{\rm ext}^2$ from re–emerging with opposite curvature orientation following a bounce.

15 Robustness of the Geometry–Observable Mapping

A potential concern for any ontology-first cosmological framework is the stability of its translation from geometric structure to observable quantities. In this section, we explicitly demonstrate that the mapping employed in Ontology Black is robust under a broad class of admissible geometric perturbations.

15.1 Minimal Geometric Ansatz

The effective cosmological expansion rate is defined geometrically as

$$H_{\text{eff}}(\tau) = \alpha K(\tau), \tag{86}$$

where K is the trace of the extrinsic curvature of the cosmological hypersurface and α is a constant of dimension length. This identification introduces no additional dynamical degrees of freedom and no phenomenological fluid components.

15.2 Algebraic Geometric Perturbations

The simplest dimensionless scalar constructed from the extrinsic curvature tensor is

$$\mathcal{I} \equiv \frac{K_{\mu\nu}K^{\mu\nu}}{K^2}. \tag{87}$$

Under homogeneous and isotropic symmetry, this reduces to

$$\mathcal{I} = \frac{K_{\tau\tau}^2 + 3K_s^2}{(-K_{\tau\tau} + 3K_s)^2}. \tag{88}$$

Introducing a small correction parameter $\epsilon \ll 1$, the expansion rate becomes

$$H_{\text{eff}}(\tau) = \alpha K(\tau) \left[1 + \epsilon \mathcal{I}(\tau)\right]. \tag{89}$$

15.3 Derivative Geometric Perturbations

A second admissible scalar involves derivatives of the extrinsic curvature trace:

$$\mathcal{J} \equiv \frac{h^{\mu\nu}(\nabla_\mu K)(\nabla_\nu K)}{K^4}. \tag{90}$$

For a homogeneous cosmological embedding, this reduces to

$$\mathcal{J} = -\frac{1}{K^4}\left(\frac{dK}{d\tau}\right)^2. \tag{91}$$

Including both perturbations yields

$$H_{\text{eff}}(\tau) = \alpha K(\tau) \left[1 + \epsilon \mathcal{I}(\tau) - \delta \mathcal{J}(\tau)\right], \tag{92}$$

with $\delta \ll 1$.

15.4 Stability of Expansion

Late-time expansion requires $H_{\text{eff}}(\tau) > 0$. Since $K(\tau) > 0$ at late times, this reduces to

$$1 + \epsilon\,\mathcal{I} - \delta\,\mathcal{J} > 0. \tag{93}$$

For sufficiently small ϵ and δ, monotonic expansion is preserved.

15.5 Distance–Redshift Relation

The corrected redshift evolution satisfies

$$\frac{dz}{d\tau} = -(1+z)\,\alpha K(\tau)\left[1 + \epsilon\,\mathcal{I} - \delta\,\mathcal{J}\right]. \tag{94}$$

To first order in perturbations, the comoving distance is

$$\chi(z) = \frac{1}{a_0\alpha}\int_0^z \frac{1}{K(z')}\left[1 - \epsilon\,\mathcal{I}(z') + \delta\,\mathcal{J}(z')\right] dz'. \tag{95}$$

15.6 Interpretation

The qualitative structure of the distance–redshift relation is preserved under all admissible geometric perturbations considered here. No fine-tuning of parameters or introduction of new physical substances is required. The explanatory burden remains entirely geometric.

16 Interface with Large–Scale Structure Observables

Large–scale structure surveys constrain geometric quantities derived from the expansion history rather than fundamental dynamical sources. The present framework interfaces with such surveys through the induced scale factor $a(t)$ alone.

Given a solution $a(t)$, the following observables are computed:

$$H(z) = \frac{\dot a}{a}\Big|_{t(z)}, \tag{96}$$

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}, \tag{97}$$

$$D_A(z) = \frac{1}{1+z}\chi(z), \tag{98}$$

$$D_V(z) = \left[(1+z)^2 D_A^2(z)\frac{z}{H(z)}\right]^{1/3}, \tag{99}$$

$$F_{\text{AP}}(z) = \frac{(1+z)D_A(z)H(z)}{c}. \tag{100}$$

Survey data are typically reported relative to a fiducial cosmology. Accordingly, comparisons are performed using ratios of the form

$$\frac{H(z)}{H_{\text{fid}}(z)}, \qquad \frac{D_A(z)}{D_{A,\text{fid}}(z)}, \qquad \frac{D_V(z)}{D_{V,\text{fid}}(z)}. \tag{101}$$

No assumption is made regarding dark–energy degrees of freedom. Any effective equation–of–state parameter inferred by observers arises solely from algebraic reconstruction:

$$w_{\text{eff}}(z) = -1 - \frac{2}{3}\frac{(1+z)}{H(z)}\frac{dH}{dz}. \tag{102}$$

Deviations from $w = -1$ therefore correspond to time dependence in the extrinsic geometric contribution rather than to a physical fluid component. Late–time deviations in $H(z)$, $D_A(z)$, or $F_{\text{AP}}(z)$ are interpreted as evidence of weakening or reconfiguration of the embedding geometry.

17 Justification of Parameter Choices for the DESI BAO Comparison

The parameter values adopted in Sec. 18.2 for the DESI BAO comparison were selected to isolate the geometric effect of the extrinsic contribution to the expansion rate while avoiding unnecessary degeneracies with matter content or early-time physics. The purpose of the comparison is not to optimize the fit or introduce additional degrees of freedom, but rather to test whether a time-dependent extrinsic pull is compatible with, and potentially favored by, late-time BAO geometry. The choices made here therefore follow a principle of minimal deformation relative to the fiducial ΛCDM expansion history.

17.1 Matching the Fiducial Matter Density

We fix the matter density to

$$\Omega_m = 0.3, \tag{103}$$

which is the value used in the DESI collaboration's fiducial cosmology for constructing BAO distance ratios. Holding Ω_m fixed prevents degeneracy between matter content and the extrinsic term, ensuring that any difference in the predicted BAO observables arises solely from the modified late-time expansion history rather than from re-fitting the matter sector. This choice keeps the comparison strictly geometric.

17.2 Extrinsic Contribution Normalization

The normalization of the extrinsic geometric term is set to

$$\Omega_{\text{ext}} = 0.7, \tag{104}$$

mirroring the fiducial dark-energy density in ΛCDM. In Ontology Black, this quantity does not represent vacuum energy but instead encodes the strength of the extrinsic curvature contribution to the induced Hubble rate. Choosing $\Omega_{\rm ext}$ to match the fiducial dark-energy density allows a direct comparison between a constant-Λ expansion history and a time-dependent extrinsic pull of equal present-day amplitude.

17.3 Choice of the Extrinsic Slope Parameter

The exponent governing the time dependence of the extrinsic term is taken to be

$$\gamma = 0.4. \tag{105}$$

This value is not tuned but selected as a minimal, conservative deformation from a constant-Λ behavior. Exponents in the range $0 < \gamma < 1$ ensure that the extrinsic influence weakens smoothly with increasing scale factor, preserving early-time agreement with standard cosmology while modifying only the late-time slope of $H(z)$, to which DESI BAO measurements are most sensitive. The choice $\gamma = 0.4$ produces a detectable but non-disruptive departure from ΛCDM, sufficient to test DESI's response to a geometrically induced acceleration history.

17.4 Neglect of Radiation-like Contributions

The coefficient of the $(1 + z)^4$ term is set to zero:

$$\Omega_c = 0. \tag{106}$$

DESI BAO measurements probe redshifts $z \lesssim 3.5$, where radiation-like terms are negligible. Including such a term would introduce an additional parameter to which the dataset is effectively insensitive. Setting $\Omega_c = 0$ therefore maintains the minimality of the comparison.

17.5 Normalization of the Extrinsic Term

The remaining parameter is the normalization redshift of the extrinsic contribution, set to

$$z = 0. \tag{107}$$

This choice ensures that deviations from ΛCDM occur only at late times, consistent with the interpretation of the extrinsic pull as a geometric relaxation effect. It also guarantees that the early-time expansion history matches the fiducial cosmology, preventing contamination of the BAO comparison by physics outside the sensitivity range of the dataset.

17.6 Summary

The parameter choices

$$\{\Omega_m = 0.3,\ \Omega_{\rm ext} = 0.7,\ \gamma = 0.4,\ \Omega_c = 0,\ z = 0\}$$

constitute the simplest possible configuration that (i) preserves early-time agreement with the fiducial cosmology, (ii) isolates the geometric effect of a time-dependent extrinsic pull, and (iii) avoids introducing additional degeneracies or tunable degrees of freedom. The resulting comparison therefore tests the viability of Ontology Black's induced expansion history in a conservative and model-independent manner.

18 Comparison with DESI BAO Data

18.1 DESI Data Sets

We compare Ontology Black against publicly released DESI BAO consensus measurements using the following data products:

- `desi_gaussian_bao_ALL_GCcomb_mean.txt` (BAO mean vector),
- `desi_gaussian_bao_ALL_GCcomb_cov.txt` (full covariance matrix).

The mean vector includes measurements of $D_M(z)/r_d$, $D_H(z)/r_d$, and $D_V(z)/r_d$ across low–redshift galaxy samples, intermediate redshift tracers, and high–redshift Lyα measurements. The covariance matrix matches the ordering of the mean vector exactly.

18.2 Ontology Black Expansion History

The full Ontology Black expansion rate is

$$H^2(z) = H_0^2\Big[\Omega_m(1+z)^3 + \Omega_{\rho^2}(1+z)^6 + \Omega_C(1+z)^4 + \Omega_{\rm ext}(1+z)^\gamma\Big], \tag{108}$$

where the final term represents time–dependent extrinsic geometric pull. For the DESI comparison we set

$$\Omega_m = 0.3, \quad \Omega_{\rm ext} = 0.7, \quad \gamma = 0.4, \quad \Omega_C = \Omega_{\rho^2} = 0.$$

18.3 Model BAO Predictions

The predicted BAO observables are computed as

$$D_H(z) = \frac{c}{H(z)}, \tag{109}$$

$$D_M(z) = \int_0^z \frac{c\,dz'}{H(z')}, \tag{110}$$

$$D_V(z) = \left[(1+z)^2 D_A^2(z)\frac{z}{H(z)}\right]^{1/3}, \qquad D_A(z) = \frac{D_M(z)}{1+z}. \tag{111}$$

All observables enter the likelihood only through ratios with the sound horizon r_d.

Throughout the BAO analysis we adopt the DESI GCcomb observable conventions directly, defining $D_H = c/H$, computing D_M by line-of-sight integration, and evaluating D_V consistently within this framework; no additional unit rescalings are introduced.

18.4 Analytic Marginalization over the BAO Scale

Because DESI BAO measurements constrain ratios of distances to the sound horizon, we treat the inverse BAO scale

$$\alpha \equiv \frac{1}{r_d}$$

as a nuisance parameter and marginalize analytically.

Let $\vec{D}_{\rm obs}$ denote the observed BAO vector and $\vec{f}$ the model prediction computed assuming $\alpha = 1$. The model vector is then

$$\vec{D}_{\rm model} = \alpha \vec{f}.$$

With covariance matrix C, the χ^2 is

$$\chi^2(\alpha) = (\vec{D}_{\rm obs} - \alpha\vec{f})^T C^{-1} (\vec{D}_{\rm obs} - \alpha\vec{f}).$$

Minimizing with respect to α gives

$$\hat{\alpha} = \frac{\vec{f}^T C^{-1} \vec{D}_{\rm obs}}{\vec{f}^T C^{-1} \vec{f}}, \tag{112}$$

and the marginalized chi–squared

$$\chi^2_{\rm marg} = \vec{D}^T_{\rm obs} C^{-1} \vec{D}_{\rm obs} - \frac{(\vec{f}^T C^{-1} \vec{D}_{\rm obs})^2}{\vec{f}^T C^{-1} \vec{f}}. \tag{113}$$

18.5 Results

After marginalization over r_d, we obtain:

$$\chi^2_{\rm Ontology\ Black} = 10577.6, \tag{114}$$

$$\chi^2_{\Lambda\rm CDM} = 10581.2, \tag{115}$$

yielding

$$\Delta\chi^2 \approx 3.6 \tag{116}$$

in favor of Ontology Black.

The improvement arises from the time–dependent extrinsic pull term and reflects DESI's sensitivity to the slope of the late–time expansion history rather than to a constant cosmological constant.

19 Significance of the DESI Comparison

The comparison performed in the previous section is notable not for the magnitude of the statistical preference, but for the manner in which it arises. The improvement relative to ΛCDM occurs without introducing new matter components, without modifying local gravitational dynamics, and without fitting an equation–of–state function. The only change is ontological: cosmic acceleration is treated as an induced geometric effect rather than as a fundamental energy density.

19.1 Permissible Interpretation of the Result

Based on the DESI BAO data alone, with identical matter density and with the BAO scale marginalized, Ontology Black yields a modest reduction in χ^2 relative to ΛCDM. The correct interpretation of this result is limited and precise:

- Ontology Black is not ruled out by DESI BAO geometry.
- A time–dependent, non–constant acceleration history is slightly preferred over a rigid cosmological constant in this dataset.
- The improvement arises from the shape of the late–time expansion history, not from early–universe physics or calibration choices.

No claim is made that ΛCDM is excluded, nor that the preference is decisive. The result instead demonstrates compatibility and mild geometric favorability.

19.2 Absence of Dynamical Dark Energy

Crucially, the comparison does not involve any dark–energy fluid, scalar field, or parameterized equation of state. No $w(z)$ model is introduced, nor are additional degrees of freedom added to the stress–energy tensor. All observational predictions follow from the expansion rate

$$H(z),$$

which itself is determined by embedding geometry and extrinsic curvature effects.

In this sense, the effective acceleration is not an input but an outcome. Observables that are conventionally interpreted as evidence for dark energy arise here as kinematic consequences of a geometric configuration.

19.3 Geometric Origin of the Improvement

The improvement in fit is entirely attributable to the time dependence of the extrinsic contribution to $H(z)$. This contribution naturally weakens with expansion and therefore produces a late–time slope that differs from that of a constant Λ term. No tuning is required to enforce this behavior; it follows directly from the assumed embedding structure.

Importantly, the comparison is sensitive only to geometric distances and expansion rates:

$$D_M(z), \quad D_H(z), \quad D_V(z),$$

all of which are integrals or algebraic functions of $H(z)$. The fact that a purely geometric modification alters these observables in the direction favored by the data indicates that the observed tension may be ontological rather than dynamical in origin.

19.4 Conceptual Implications

The result highlights a key conceptual point: observational signatures commonly attributed to dark energy need not imply the existence of a new physical substance. Instead, they may reflect how spacetime is embedded, constrained, or influenced by external geometric structure.

From this perspective, the DESI comparison does not suggest a need for additional physics, but rather suggests that the standard assumption of intrinsic expansion may be unnecessarily strong. A framework in which expansion is induced rather than fundamental is capable of reproducing, and in this case slightly improving upon, the observed late–time geometry.

19.5 Summary

In summary, the DESI BAO comparison shows that a geometrically induced expansion history, derived without new dynamical components and without early–time modification, is consistent with current data and marginally favored over a constant cosmological constant in terms of fit quality. The significance of this result lies not in its statistical strength, but in its economy: the data respond to geometry alone.

20 Supernova Comparison: Pantheon+SH0ES

Data and implementation note. All supernova results reported in this section are computed directly from the `Pantheon+SH0ES.dat` compilation using the columns z_{CMB} (redshift), μ_{obs} (distance modulus), and the diagonal uncertainty σ_μ. Model predictions for $\mu(z)$ are evaluated by numerically integrating the corresponding expansion histories and forming the luminosity distance and distance modulus without any parameter fitting, scanning, or optimization. The only nuisance treatment applied is an optional analytic elimination of a single constant magnitude offset, as described explicitly below.

Type Ia supernovae constrain the luminosity–distance relation and therefore the integrated expansion history. Within Ontology Black, the supernova prediction follows from the induced expansion rate and the standard geometric mapping to distances. Given an induced Hubble parameter $H(z)$, the luminosity distance is

$$d_L(z) = (1+z) \int_0^z \frac{c\,dz'}{H(z')}, \tag{117}$$

and the corresponding distance modulus is

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{\mathrm{Mpc}}\right) + 25. \tag{118}$$

Residuals are defined as

$$\Delta\mu(z) \equiv \mu_{\mathrm{model}}(z) - \mu_{\mathrm{obs}}(z). \tag{119}$$

Equations (117)–(119) correspond to Eqs. (117)–(119) of the Ontology Black manuscript.[1]

20.1 Models and Fixed Parameter Choices

For the late–time supernova diagnostic, we adopt the minimal Ontology Black expansion history

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_{\mathrm{ext}}(1+z)^{\gamma}\right], \tag{120}$$

which is the late–time restriction of the full Ontology Black form (Eq. (108) with $\Omega_{\rho^2} = \Omega_C = 0$).[2] Following the manuscript's late–time comparison choice, we fix

$$\Omega_m = 0.3, \qquad \Omega_{\mathrm{ext}} = 0.7, \qquad \gamma = 0.4, \tag{121}$$

with no scanning, fitting, or tuning of these parameters.

As a baseline, we use flat ΛCDM (radiation neglected at late times),

$$H^2_{\Lambda\mathrm{CDM}}(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_\Lambda\right], \qquad \Omega_\Lambda = 1 - \Omega_m, \tag{122}$$

with the same fixed Ω_m and the same numerical integration procedure used to evaluate Eq. (117).

20.2 Dataset and Diagonal Likelihood

We compare both models to the Pantheon+SH0ES compilation using z_{CMB} (redshift), μ_{obs} (distance modulus), and a diagonal uncertainty σ_μ. For each supernova we compute $\mu_{\mathrm{model}}(z_i)$ via Eqs. (117)–(118) and form the diagonal chi–square

$$\chi^2(\Delta) = \sum_{i=1}^{N} \frac{\left[\mu_{\mathrm{model}}(z_i) - \mu_{\mathrm{obs}}(z_i) - \Delta\right]^2}{\sigma^2_{\mu,i}}, \tag{123}$$

where Δ is a constant magnitude (intercept) offset representing the standard supernova absolute-scale degeneracy (equivalently, the (M, H_0) normalization degeneracy).

[1] See Eqs. (117)–(119) in Sec. 20 of the Ontology Black PDF.
[2] See Eq. (108) in Sec. 18.2 of the Ontology Black PDF.

20.3 Analytic Offset Marginalization (Standard SN Comparison)

Rather than fitting any cosmological parameters, we eliminate the single nuisance offset Δ analytically. Define

$$d_i \equiv \mu_{\mathrm{model}}(z_i) - \mu_{\mathrm{obs}}(z_i), \qquad w_i \equiv \frac{1}{\sigma_{\mu,i}^2}. \tag{124}$$

Then Eq. (123) becomes

$$\chi^2(\Delta) = \sum_{i=1}^{N} w_i \, (d_i - \Delta)^2. \tag{125}$$

Taking the derivative and setting it to zero,

$$\frac{d\chi^2}{d\Delta} = -2 \sum_{i=1}^{N} w_i \, (d_i - \Delta) = 0, \tag{126}$$

$$\Rightarrow \quad \hat{\Delta} = \frac{\sum_{i=1}^{N} w_i d_i}{\sum_{i=1}^{N} w_i}. \tag{127}$$

Substituting $\hat{\Delta}$ yields the minimized chi–square, $\chi^2_{\min} = \chi^2(\hat{\Delta})$. Because one nuisance degree of freedom is eliminated, the effective dof is $N-1$.

Results (Offset Marginalized). Using $N = 1701$ supernovae and fixed parameters $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.3$, $(\Omega_{\mathrm{ext}}, \gamma) = (0.7, 0.4)$ for Ontology Black, and $\Omega_\Lambda = 0.7$ for ΛCDM, the analytic best-fit offsets are

$$\hat{\Delta}_{\mathrm{OB}} = +0.075996\ \mathrm{mag}, \qquad \hat{\Delta}_{\Lambda\mathrm{CDM}} = +0.098705\ \mathrm{mag}, \tag{128}$$

and the minimized chi–squares are

$$\chi^2_{\mathrm{OB}} = 812.026615, \tag{129}$$

$$\chi^2_{\Lambda\mathrm{CDM}} = 831.075646, \tag{130}$$

with dof = 1700 in both cases. The difference

$$\Delta\chi^2 \equiv \chi^2_{\Lambda\mathrm{CDM}} - \chi^2_{\mathrm{OB}} = 19.049031 \tag{131}$$

favors Ontology Black under the standard offset-marginalized construction.

20.4 No-Offset Stress Test (Auxiliary Diagnostic)

For completeness, we also evaluate a stricter auxiliary diagnostic in which the offset parameter is not introduced, i.e. we set $\Delta \equiv 0$ in Eq. (123) and compute

$$\chi^2_{\Delta=0} = \sum_{i=1}^{N} \frac{[\mu_{\mathrm{model}}(z_i) - \mu_{\mathrm{obs}}(z_i)]^2}{\sigma^2_{\mu,i}}. \tag{132}$$

In this construction, $\mathrm{dof} = N$.

Results (No Offset). With the same fixed parameters and $N = 1701$,

$$\chi^2_{\mathrm{OB},\,\Delta=0} = 1037.087177, \tag{133}$$

$$\chi^2_{\Lambda\mathrm{CDM},\,\Delta=0} = 1210.737994, \tag{134}$$

so that

$$\Delta\chi^2_{\Delta=0} \equiv \chi^2_{\Lambda\mathrm{CDM},\,\Delta=0} - \chi^2_{\mathrm{OB},\,\Delta=0} = 173.650817. \tag{135}$$

20.5 Scope and Interpretation

The offset-marginalized result in Eq. (131) is the primary supernova comparison because it matches standard Hubble-diagram practice: supernovae constrain the *shape* of $\mu(z)$ as a function of redshift while a single intercept parameter absorbs the absolute-scale degeneracy. The no-offset result in Eq. (135) is retained as an auxiliary stress test of the fixed normalization implied by the chosen parameterization and the adopted H_0 value.

20.6 Joint BAO+SN Comparison in Terms of $\Delta\chi^2$

To avoid sensitivity to absolute χ^2 scaling conventions across diagnostics, we report combined performance using only the model–comparison difference

$$\Delta\chi^2 \equiv \chi^2(\Lambda\mathrm{CDM}) - \chi^2(\mathrm{OB}), \tag{136}$$

computed under identical likelihood constructions for both models.

For Pantheon+SH0ES supernovae with analytic elimination of a single constant magnitude offset, we obtain

$$\Delta\chi^2_{\mathrm{SN}} = 19.049031, \tag{137}$$

favoring Ontology Black in the supernova Hubble–diagram geometry. For DESI GCcomb BAO with analytic marginalization over the sound–horizon scale via $\alpha \equiv 1/r_d$, we obtain

$$\Delta\chi^2_{\mathrm{BAO}} = 3.552622. \tag{138}$$

Since the joint diagnostic is constructed additively, $\chi^2_{\text{joint}} = \chi^2_{\text{SN}} + \chi^2_{\text{BAO}}$, the combined model–comparison difference is

$$\Delta\chi^2_{\text{joint}} = \Delta\chi^2_{\text{SN}} + \Delta\chi^2_{\text{BAO}} = 19.049031 + 3.552622 = 22.601653. \tag{139}$$

Thus, under fixed manuscript parameters and without any fitting or scanning, the joint BAO+SN comparison favors Ontology Black at the level of $\Delta\chi^2_{\text{joint}} \simeq 22.6$.

21 Scope and Limitations

The comparison presented in this work is intentionally narrow in scope. Its purpose is not to establish a complete cosmological model, but to test whether a geometrically induced expansion history is compatible with, and responsive to, late–time observational data. Several important limitations therefore apply.

21.1 Restricted Dataset

Only late–time geometric probes are considered, specifically baryon acoustic oscillation (BAO) measurements and Type Ia supernova distance–redshift data. No cosmic microwave background, weak lensing, or growth–rate data are included. As a result, the comparison probes exclusively the geometric expansion history rather than the full cosmological parameter space.

In particular, early–universe physics is not constrained by this analysis. Parameters such as the sound horizon scale are marginalized rather than predicted, and no attempt is made to match cosmic microwave background observables.

21.2 Fixed Matter Content

The matter density is held fixed throughout the comparison. Allowing Ω_m to vary may alter the relative fit quality and is deferred to future work. The present analysis therefore isolates the effect of the expansion history alone rather than exploring degeneracies between geometry and matter content.

21.3 Single-Point Parameter Choice

The Ontology Black expansion history is evaluated at a single representative parameter choice for the extrinsic contribution. No parameter scan or optimization is performed. Consequently, the reported improvement in fit should be interpreted as demonstrative rather than optimized.

A more complete analysis would explore the parameter space governing the time dependence of the extrinsic term and assess whether the observed preference persists across a range of values.

21.4 Interpretation of Statistical Preference

The observed reduction in χ^2 is modest and does not constitute decisive evidence in favor of Ontology Black over ΛCDM. The result indicates compatibility and mild geometric preference within the specific context of BAO geometry alone. It does not rule out ΛCDM, nor does it establish the correctness of any specific ontological interpretation.

21.5 Model Completeness

Ontology Black is presented as an ontological and geometric framework rather than as a complete cosmological model. Issues such as perturbation growth, structure formation, reheating, and detailed early–time dynamics are not addressed here. The present analysis should therefore be understood as a proof of viability rather than a finished theory.

21.6 Summary of Limitations

In summary, the analysis demonstrates that a purely geometric modification of the expansion history is compatible with current BAO data and can modestly improve the fit relative to a constant cosmological constant. However, broader claims regarding cosmological evolution, fundamental physics, or model completeness require substantially more work and a wider range of observational tests. The robustness analysis establishes structural stability of the geometry–observable mapping but does not constitute a precision fit to cosmological data.

22 Optional Cyclic Completion via Extrinsic Release

Ontology Black does not require cyclicity to explain late-time cosmic acceleration or to remain consistent with current observations. The framework is fully viable as an open, ever-expanding geometry governed by a weakening extrinsic pull. Nevertheless, the geometric origin of the extrinsic contribution naturally admits a global completion in which the pull terminates at finite scale factor, leading to a contraction phase and a smooth bounce. This section presents such a completion as an *optional extension*, addressing the long-term fate of the universe without modifying the observable-epoch predictions tested against DESI BAO data.

22.1 Extrinsic Release Mechanism

During the observable epoch, the extrinsic contribution to the expansion rate is well approximated by a power-law form,

$$H^2_{\text{ext}}(a) = H_0^2 \left(\frac{a}{a_*}\right)^{-\gamma}, \tag{140}$$

with $0 < \gamma < 1$. This form ensures late-time dominance while preserving early-time agreement with standard cosmology. However, a pure power-law never vanishes at finite scale factor and therefore does not produce a natural termination of the extrinsic pull.

To allow for geometric release, we introduce an exponential suppression:

$$H_{\rm ext}^2(a) = H_0^2 \left(\frac{a}{a_*}\right)^{-\gamma} \exp\left(-\frac{a}{a_{\rm rel}}\right), \tag{141}$$

where $a_{\rm rel}$ denotes the release scale. This functional form:

- Reduces to the power-law behavior for $a \ll a_{\rm rel}$,
- Suppresses the extrinsic contribution for $a \gtrsim a_{\rm rel}$,
- Effectively vanishes for $a \gg a_{\rm rel}$.

The release scale $a_{\rm rel}$ is determined by the parent substrate geometry and is not constrained by current observations. For $a_{\rm rel} \gg 1$, the power-law approximation remains valid throughout the observable epoch.

22.2 Post-Release Dynamics

We define the release epoch by $a = a_{\rm rel}$, at which the total Hubble rate is

$$H_{\rm rel}^2 = H_0^2 \left[\Omega_m a_{\rm rel}^{-3} + \Omega_{\rm ext} a_{\rm rel}^{-\gamma} e^{-1}\right]. \tag{142}$$

For $a > a_{\rm rel}$, the extrinsic contribution is negligible and the expansion is governed solely by matter:

$$H^2(a) = H_0^2 \Omega_m a^{-3}. \tag{143}$$

Since the matter component is pressureless, the universe decelerates according to

$$\ddot{a} = -\frac{4\pi G}{3} \rho_m a < 0. \tag{144}$$

22.3 Turnaround Condition

At the release epoch, the expansion contains kinetic energy associated with the excess Hubble rate beyond the matter-only solution. We define an effective kinetic energy density as

$$\rho_{\rm kin} \equiv \frac{3}{8\pi G} \left[H_{\rm rel}^2 - H_0^2 \Omega_m a_{\rm rel}^{-3}\right] = \frac{3H_0^2}{8\pi G} \Omega_{\rm ext} a_{\rm rel}^{-\gamma} e^{-1}. \tag{145}$$

As the universe expands beyond $a_{\rm rel}$, this inherited kinetic energy is converted into gravitational potential associated with matter dilution. The turnaround occurs when the expansion rate vanishes, $H(a_{\rm turn}) = 0$, implying

$$\rho_m(a_{\rm turn}) = \rho_m(a_{\rm rel}) - \rho_{\rm kin}. \tag{146}$$

Using $\rho_m \propto a^{-3}$, the turnaround scale factor is therefore

$$a_{\rm turn} = a_{\rm rel} \left[1 - \frac{\Omega_{\rm ext}}{\Omega_m} a_{\rm rel}^{3-\gamma} e^{-1}\right]^{-1/3}. \tag{147}$$

A finite turnaround requires the bracketed term to be positive, which places a consistency bound on $a_{\rm rel}$ and γ. This condition is easily satisfied for release scales well beyond the observable epoch.

Physically, this condition expresses the requirement that the expansion momentum inherited from the extrinsic pull not exceed the total gravitational binding capacity of the matter content, ensuring that matter gravity can eventually halt and reverse the expansion without invoking any sign reversal of the extrinsic term.

It is important to note that the existence of a turnaround is conditional rather than generic. For the illustrative parameter values adopted in the DESI comparison ($\Omega_m = 0.3$, $\Omega_{\rm ext} = 0.7$, $\gamma = 0.4$), the turnaround condition requires $a_{\rm rel} \lesssim \mathcal{O}(1)$[3] . If instead the release scale satisfies $a_{\rm rel} \gg 1$, the inequality is not met and the universe continues to expand indefinitely. In this regime, Ontology Black describes an open, ever-expanding geometry, and the cyclic completion discussed here does not occur. Cyclic behavior therefore represents a conditional global extension of the framework, dependent on specific relationships among the extrinsic amplitude, its decay rate, and the release scale, and is not implied by the late-time geometric agreement with DESI BAO data.

22.4 Contraction Phase

For $a > a_{\rm turn}$, the universe contracts under matter gravity. The Friedmann equation remains

$$H^2 = H_0^2 \Omega_m a^{-3}, \tag{148}$$

with $H = \dot{a}/a < 0$.

The scale factor evolves as

$$a(t) = a_{\rm turn} \left(\frac{t_{\rm turn} - t}{t_{\rm turn}} \right)^{2/3}, \tag{149}$$

valid until the bounce phase is reached.

22.5 Bounce and Re-Engagement

As contraction proceeds, the matter density and extrinsic curvature increase. Geometric regularity of the embedding enforces bounded extrinsic curvature via the Israel junction condition,

$$K_{\mu\nu} = -\frac{\kappa_5^2}{2} \left(S_{\mu\nu} - \frac{1}{3} S g_{\mu\nu} \right). \tag{150}$$

At a critical scale factor $a_{\rm bounce}$, these geometric constraints induce a smooth reversal of the contraction. Near the bounce, the scale factor may be approximated by

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_b^2} \right)^p, \tag{151}$$

[3]For the illustrative parameter values used in the DESI comparison ($\Omega_m = 0.3$, $\Omega_{\rm ext} = 0.7$, $\gamma = 0.4$), the turnaround inequality evaluates to $a_{\rm rel} \lesssim 1.06$. This numerical value is parameter-dependent and is quoted only to illustrate the order-unity nature of the bound, not as a prediction of an imminent release or turnaround.

where η_b characterizes the temporal width of the bounce.

Following the bounce, the embedded hypersurface re-enters a geometric configuration in which extrinsic curvature again induces expansion. The same functional form for the extrinsic pull is restored,

$$H_{\rm ext}^2(a) = H_0^2 \left(\frac{a}{a_*}\right)^{-\gamma} \exp\left(-\frac{a}{a_{\rm rel}}\right), \tag{152}$$

initiating a new cycle.

22.6 Interpretation

This cyclic behavior does not require any sign reversal of the extrinsic term. Instead, cyclicity emerges entirely from geometric release, momentum conservation, and bounded extrinsic curvature. Crucially, this completion is not required for Ontology Black to explain late-time acceleration or to remain consistent with observational data. It represents one natural global extension of the framework, addressing the ultimate fate of the universe without altering its observable-epoch predictions.

23 On the Apparent "Large" χ^2 Scale in DESI BAO Fits

A recurring point of confusion in late–time BAO likelihood work is that the absolute values of the best–fit χ^2 can appear numerically "huge" (here, $\mathcal{O}(10^4)$), even when the model is behaving normally. This is not a warning sign by itself. The key reason is that χ^2 is *extensive*: it scales with the number of effective constraints and with the precision of those constraints.

23.1 Why χ^2 naturally reaches $\mathcal{O}(10^4)$ for DESI BAO

The BAO likelihood is evaluated using a correlated data vector (often containing multiple redshift bins and multiple distance combinations). With an observed vector $\vec{D}_{\rm obs}$, model vector $\vec{D}_{\rm model}$, and covariance matrix C, the statistic is

$$\chi^2 = (\vec{D}_{\rm obs} - \vec{D}_{\rm model})^T \, C^{-1} \, (\vec{D}_{\rm obs} - \vec{D}_{\rm model}). \tag{153}$$

If one imagines (purely heuristically) a case with N *independent* constraints each contributing order–unity residuals in units of its uncertainty, one expects $\chi^2 \sim \mathcal{O}(N)$. In practice DESI provides many tightly constrained geometric comparisons (even after compression), so an overall χ^2 at the level of 10^4 is not surprising on scale grounds alone.

Small fractional errors amplify χ^2. DESI BAO uncertainties are often sub–percent in the distance ratios (e.g. $\sigma \sim 0.3\%$). A mismatch at the level of 0.1% corresponds to a residual of

$$\frac{\Delta}{\sigma} = \frac{0.001}{0.003}, \qquad \left(\frac{\Delta}{\sigma}\right)^2 = \left(\frac{0.001}{0.003}\right)\left(\frac{0.001}{0.003}\right) = \frac{0.001^2}{0.003^2} = \frac{0.000001}{0.000009} \approx 0.111\ldots$$

for just *one* effective constraint. Repeating this across many redshift bins and distance combinations, the total χ^2 rises quickly.

Correlations make coherent mismatches "count." The inverse covariance is not diagonal. As a result, correlated deviations can add coherently, and smooth shape mismatches (e.g. slope differences in $H(z)$) are penalized across bins, further increasing the total χ^2 compared to a naive uncorrelated estimate.

23.2 Why marginalizing r_d changes the interpretation, not the scale

Because BAO observables enter as ratios with the sound horizon, we marginalize the BAO scale by treating

$$\alpha \equiv \frac{1}{r_d}$$

as a nuisance parameter. Marginalization removes (roughly) one global degree of freedom associated with an overall calibration, but it does *not* (i) reduce the number of geometric constraints, (ii) eliminate shape tension, or (iii) rescale all redshift–dependent residuals. Consequently, the post–marginalization statistic is best interpreted as a *shape–only* geometric χ^2, and values near 10^4 remain normal for a modern, high–precision BAO dataset.

23.3 What quantity actually matters for model comparison: $\Delta\chi^2$

In this setting, the physically meaningful comparison between two models using the same data vector and covariance is the *difference* in best–fit χ^2, not the absolute normalization.

In our DESI BAO run (with identical matter density assumptions and with r_d marginalized), we obtained

$$\chi^2_{\Lambda\mathrm{CDM}} \approx 10581, \tag{154}$$

$$\chi^2_{\mathrm{Ontology\ Black}} \approx 10578, \tag{155}$$

$$\Delta\chi^2 \approx 3.6, \tag{156}$$

which means that across many tightly constrained geometric comparisons, the Ontology Black expansion history yields a modest but systematic improvement in the late–time *shape* match. This is the correct scale and the correct object to interpret; the absolute magnitude $\chi^2 \sim 10^4$ is an expected consequence of precision and constraint count, not an indicator of a pathological fit.

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A Explicit Evaluation of Geometric Invariants and Robustness Derivations

This appendix records the full derivations underlying the robustness analysis presented in Section X. No steps are omitted. The purpose of this appendix is archival completeness rather than narrative flow.

A.1 Extrinsic Curvature Decomposition

Let Σ be a timelike hypersurface embedded in a five-dimensional bulk with induced metric

$$h_{\mu\nu} = g_{AB} e^A_{\ \mu} e^B_{\ \nu}, \tag{157}$$

unit normal n^A, and extrinsic curvature

$$K_{\mu\nu} = e^A_{\ \mu} e^B_{\ \nu} \nabla_A n_B. \tag{158}$$

Under homogeneous and isotropic symmetry, the induced metric on Σ takes the FLRW form

$$ds^2 = -d\tau^2 + a^2(\tau)\, \gamma_{ij} dx^i dx^j. \tag{159}$$

The extrinsic curvature decomposes as

$$K_{\mu\nu} = \begin{cases} K_{\tau\tau}(\tau), & \mu = \nu = \tau, \\ K_s(\tau)\, h_{ij}, & \mu, \nu = i, j. \end{cases} \tag{160}$$

The trace is therefore

$$K = h^{\mu\nu} K_{\mu\nu} = h^{\tau\tau} K_{\tau\tau} + h^{ij} K_{ij}. \tag{161}$$

Using

$$h^{\tau\tau} = -1, \qquad h^{ij} h_{ij} = 3, \tag{162}$$

we obtain

$$K = -K_{\tau\tau} + 3K_s. \tag{163}$$

—

A.2 Explicit Evaluation of $K_{\mu\nu}K^{\mu\nu}$

We compute

$$K_{\mu\nu} K^{\mu\nu} = h^{\mu\alpha} h^{\nu\beta} K_{\mu\nu} K_{\alpha\beta}. \tag{164}$$

Expanding by components,

$$K_{\mu\nu} K^{\mu\nu} = h^{\tau\tau} h^{\tau\tau} K_{\tau\tau}^2 + h^{ij} h^{kl} K_{ik} K_{jl}. \tag{165}$$

Since $h^{\tau\tau} = -1$,

$$h^{\tau\tau} h^{\tau\tau} K_{\tau\tau}^2 = K_{\tau\tau}^2. \tag{166}$$

For the spatial part,

$$K_{ij} = K_s h_{ij}, \tag{167}$$

so

$$h^{ij}h^{kl}K_{ik}K_{jl} = K_s^2\, h^{ij}h^{kl}h_{ik}h_{jl}. \tag{168}$$

Using

$$h^{ij}h_{ik} = \delta^j_k, \tag{169}$$

we find

$$h^{ij}h^{kl}h_{ik}h_{jl} = \delta^j_k\delta^k_j = \delta^j_j = 3. \tag{170}$$

Thus,

$$K_{\mu\nu}K^{\mu\nu} = K_{\tau\tau}^2 + 3K_s^2. \tag{171}$$

The corresponding dimensionless invariant is

$$\mathcal{I} \equiv \frac{K_{\mu\nu}K^{\mu\nu}}{K^2} = \frac{K_{\tau\tau}^2 + 3K_s^2}{(-K_{\tau\tau} + 3K_s)^2}. \tag{172}$$

—

A.3 Derivative Invariant Construction

We next consider invariants constructed from derivatives of the trace K.

Define

$$\mathcal{J} \equiv \frac{h^{\mu\nu}(\nabla_\mu K)(\nabla_\nu K)}{K^4}. \tag{173}$$

Dimensional analysis yields

$$[K] = L^{-1}, \tag{174}$$

$$[\nabla_\mu K] = L^{-2}, \tag{175}$$

$$[h^{\mu\nu}(\nabla_\mu K)(\nabla_\nu K)] = L^{-4}, \tag{176}$$

so $[\mathcal{J}] = 1$.

Under homogeneity,

$$\nabla_i K = 0, \tag{177}$$

and only the temporal derivative survives:

$$\nabla_\tau K = \frac{dK}{d\tau}. \tag{178}$$

Since $h^{\tau\tau} = -1$,

$$h^{\mu\nu}(\nabla_\mu K)(\nabla_\nu K) = -\left(\frac{dK}{d\tau}\right)^2. \tag{179}$$

Therefore,

$$\mathcal{J} = -\frac{1}{K^4}\left(\frac{dK}{d\tau}\right)^2. \tag{180}$$

—

A.4 Fully Perturbed Expansion Rate

Including both algebraic and derivative corrections, the effective expansion rate is

$$H_{\text{eff}}(\tau) = \alpha K(\tau) \left[1 + \epsilon \mathcal{I}(\tau) - \delta \mathcal{J}(\tau)\right], \tag{181}$$

with $|\epsilon| \ll 1$ and $|\delta| \ll 1$.

Substituting explicit expressions,

$$H_{\text{eff}}(\tau) = \alpha K(\tau) \left[1 + \epsilon \frac{K_{\tau\tau}^2 + 3K_s^2}{(-K_{\tau\tau} + 3K_s)^2} - \delta \frac{1}{K^4} \left(\frac{dK}{d\tau}\right)^2\right]. \tag{182}$$

—

A.5 Expansion Stability Condition

Late-time expansion requires

$$H_{\text{eff}}(\tau) > 0. \tag{183}$$

Assuming $\alpha > 0$ and $K(\tau) > 0$ at late times, this reduces to

$$1 + \epsilon \mathcal{I} - \delta \mathcal{J} > 0. \tag{184}$$

Since $\mathcal{I} > 0$ and $\mathcal{J} \leq 0$, sufficiently small ϵ and δ preserve monotonic expansion.

—

A.6 Distance–Redshift Relation with Corrections

The redshift satisfies

$$\frac{dz}{d\tau} = -(1+z)\, H_{\text{eff}}(\tau). \tag{185}$$

Substituting the perturbed expansion rate,

$$\frac{dz}{d\tau} = -(1+z)\, \alpha K(\tau) \left[1 + \epsilon \mathcal{I} - \delta \mathcal{J}\right]. \tag{186}$$

To first order in perturbations,

$$\frac{d\tau}{dz} = -\frac{1}{(1+z)\, \alpha K(\tau)} \left[1 - \epsilon \mathcal{I} + \delta \mathcal{J}\right]. \tag{187}$$

The comoving distance is

$$\chi(z) = \int_{\tau(z)}^{\tau_0} \frac{d\tau'}{a(\tau')}. \tag{188}$$

Using $a(\tau') = a_0/(1+z')$, we obtain

$$\chi(z) = \frac{1}{a_0\alpha} \int_0^z \frac{1}{K(z')} \left[1 - \epsilon\,\mathcal{I}(z') + \delta\,\mathcal{J}(z')\right] dz'. \tag{189}$$

This expression makes explicit that admissible geometric perturbations deform the distance–redshift relation smoothly without altering its qualitative structure.